

**PERTH COLLEGE**

**Year 12**

**Semester One Examination 2011**

**Question/Answer booklet**

**MATHEMATICS**

**SPECIALIST 3CD**

**Section Two**

**Calculator – assumed**

|  |
| --- |
| Student Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

**Time allowed for this section**

Reading time before commencing work: 10 minutes

Working time for paper: 100 minutes

**Material required/recommended for this section**

**To be provided by the supervisor**

Question/answer booklet for Section Two

Formula sheet

**To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: drawing instruments, templates, notes (both sides of two unfolded sheets of A4 paper) and up to three calculators (CAS, graphic or scientific) which satisfy the conditions set by the Curriculum Council for this course.

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor

**before** reading any further.

**Structure of this paper**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Number of questions available | Number of questions to be attempted | Working  time  (minutes) | Marks available |
| Section One  Calculator-free | 5 | 5 | 50 minutes | 40 |
| **Section Two**  **Calculator-assumed** | **11** | **11** | **100 minutes** | **80** |
| **Total marks** | | | | 120 |

**Instructions to candidates**

1. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer
   1. Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   2. Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.
2. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.
3. It is recommended that you **do not use pencil,** except in diagrams.

**Question 6** [ 3 + 2 = 5 marks]

**(a)** Find the acute angle between the plane and the line . [3]

**(b)** Find the value of so that the vector is normal to the plane . [2]

**Question 7** [2 +3 + 2 + 5 = 12 marks]

**(a)** The line passes through the points and .

Find the vector equation of line . [2]

**(b)** The line is perpendicular to line from (a), and intersects at .

Find the vector equation of line . [3]

**(c)** Find equation of the plane Π that contains both and found above. [2]

***(Question 7 continued)***

**(d)** Point is at a certain distance from both lines.

Which line, or , is the closest to point ?

Show clear evidence to support your answer. [5]

**Question 8** [6 + 2 = 8 marks]

Triangle has its vertices at and

**(a)** Find the exact value of all internal angles of triangle using the dot product. [6]

**(b)** Point divides the segment internally in the ratio .

Find the coordinates of point . [2]

**Question 9** [2 + 3 + 3 = 8 marks]

**(a)** Solve: [2]

**(b)** Use an algebraic method to solve: [3]

**(c)** Use a graphical or algebraic method to solve:

Explain your answer. [3]

**Question 10** [4 marks]

Use first principles to show that the derivative of is [4]

**Question 11** [6 marks]

A fish of species P each day consumes 8g of food A, 5g of food B and 3g of food C.

A fish of species Q each day consumes 5g of food A, 3g of food B and 2g of food C.

A fish of species R consumes 3g, 1g and 1g respectively of food A, B and C.

A given environment has 310g of food A, 170g of food B and 115g of food C.

**(a)** Use a matrix method to find the population size of the three species that will

consume exactly all of the available food in:

**(i)** one day. [2]

**(ii)** five days. [2]

**(b)** Given that there were 10 fish of each species in an environment, use a matrix method

to determine how much food of each type would be required each day. [2]

**Question 12** [5 marks]

Heat is applied to a spherical ball so that it expands while preserving its spherical shape.

When the surface area of the sphere is exactly cm2, it is increasing at the rate of 50 mm2 per second.

Find the rate of increase of the volume of the ball at this instant. [5]

**Question 13** [ 6 marks]

A curve is defined implicitly with the equation .

Find the exact equation of the tangent to this curve at the point .

Show ALL working. [6]

**Question 14** [2 + 2 + 3 + 3 + 4 = 14 marks]

Evaluate the following integrals, showing sufficient working to justify your answers.

**(a)** [2]

**(b)** [2]

**(c)** [3]

***(Question 14 continued)***

**(d)** [3]

**(e)** *(Hint: partial fractions)* [4]

**Question 15** [1 + 3 + 2 + 3 = 9 marks]

Salmon can be bred in large ponds for commercial purposes.

A particular species of salmon lives to four years of age. The survival rate of this species in their first, second and third years is 0.5%, 7% and 15% respectively. It is known that females only reproduce during their third and fourth years of age. Each 3 year old female can produce 5 000 female offspring, while each 4 year old female can produce 2 000 female offspring.

A new pond is started with 1 000 of each 1, 2, 3 and 4 year old female salmon, including an appropriate amount of males.

**(a)** Find the Leslie matrix for this population. [1]

**(b)** Showing working to support your answer, find (to the nearest whole number) the total

female population after:

**(i)** 1 year [1]

**(ii)** 2 years [1]

**(iii)** 5 years [1]

***(Question 15 continued)***

**(c)** When will the female population be more than four million again? [2]

**(d)** After 6 years the salmon population is badly affected by disease, and hence the survival

rates have decreased to 0.2%, 5% and 10% for the first, second and third years respectively.

Determine whether the population of salmon is increasing or decreasing due to the disease.

Show clear evidence to support your answer. [3]

**Question 16** [2 + 1 = 3 marks]

Consider matrix

**(a)** Show that [2]

**(b)** State the restrictions on , if any, for [1]

**end of paper**

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